Exercise 58

A manufacturer produces bolts of a fabric with a fixed width. The quantity q of this fabric (measured in yards) that is sold is a function of the selling price p (in dollars per yard), so we can write $q = f(p)$. Then the total revenue earned with selling price p is $R(p) = pf(p)$.

(a) What does it mean to say that $f(20) = 10,000$ and $f'(20) = -350$?

(b) Assuming the values in part (a), find $R'(20)$ and interpret your answer.

Solution

$f(20) = 10,000$ means that 10,000 bolts are sold if the price is \$20/yard. $f'(20) = -350$, on the other hand, gives the rate that the number of bolts sold changes with respect to an increase in price when the price is \$20 per yard.

$$R'(p) = \frac{d}{dp}[pf(p)] = f(p) + pf'(p)$$

Evaluate this derivative at $p = 20$.

$$R'(20) = f(20) + 20f'(20) = 10\,000 + 20(-350) = 3000$$

This means that the rate that revenue increases with respect to an increase in price when that price is \$20 is +\$3000.